**Homework:**

4) Singular-Value-Decomposition (SVD) (1 points)

Describe the relation between the rank of a matrix, A, and its singular values?

How Singular Value Decomposition (SVD) can be used for image compression or noise reduction?

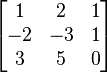
Answer:

Any matrix can be written in this form:

A=U\*∑ \*VT  ∑ is diagonal, U and V are orthogonal

This equations are always true ATA=V\*∑T\*∑\*VT  C\*V=U\*∑

The matrix

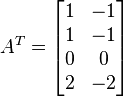


has rank 2: the first two rows are linearly independent, so the rank is at least 2, but all three rows are linearly dependent (the first is equal to the sum of the second and third) so the rank must be less than 3.

The matrix

A=\begin{bmatrix}1&1&0&2\\-1&-1&0&-2\end{bmatrix}

has rank 1: there are nonzero columns, so the rank is positive, but any pair of columns is linearly dependent. Similarly, the transpose



of A has rank 1. Indeed, since the column vectors of A are the row vectors of the transpose of A, the statement that the column rank of a matrix equals its row rank is equivalent to the statement that the rank of a matrix is equal to the rank of its transpose, i.e., rk(A) = rk(AT).

For noise reduction, let a matrix *A* represent the noisy signal, compute the SVD, and then discard small singular values of *A*. It can be shown that the small singular values mainly represent the noise, and thus the rank-*k* matrix *Ak* represents a filtered signal with less noise.

5) Karhunen-Loeve Transform (KLT) (2 points)

Briefly describe how SVD can be used to find the eigenvalues of a matrix, A?

How KLT is used to map data into a lower dimension?

Answer:

The Karhunen-Loeve transform (KLT) is defined as the linear transformation whose basis vectors are the eigenvectors of the covariance matrix of the data. As it diagonalizes the covariance matrix, it decorrelates the data.

KLT minimizes the theoretical bound on bit rate as given by the signal entropy